Question Design: IVth Semester

GMA-208(Generic): Probability Theory and Statistics

Question Type / Unit	MCQ	VSA	SA (I)	SA (II)	Essay	Mark
I	1(1)=1	2(1)=2	1(3)=3	2(4)=8	6(2)=12	26
II	1(1)=1	2(1)=2	3(3)=9	1(4)=4	6(2)=12	28
III	1(1)=1	2(1)=2	1(3)=3	2(4)=8	6(2)=12	26
	3(1)=3	6(1)=6	5(3)=15	5(4)=20	6(6)=36	**80**

Sample Question: DHANAMANJURI UNIVERSITY

Name of the Programme: B.A/B.Sc. Mathematics
Semester: Fourth
Paper code: GMA-208
Paper Title: Probability Theory and Statistics
Full Mark: 80 Pass Mark:

The figures in the margin indicate full marks for the questios
Answer all the questions

Q1. Choose and rewrite the correct answer for each of the following: $3 \times 1 = 3$

(a) If $\emptyset(t)$ is the characteristic function of a random variable X then

(i) $|\emptyset(t)| \leq 1$ (ii) $|\emptyset(t)| \leq 0$ (iii) $|\emptyset(t)| \geq 1$ (iv) $|\emptyset(t)| \leq \infty$

(b) If a random variable X is defined to have an exponential distribution with parameter $\lambda > 0$, then its mean is given by

(i) $e^{-\lambda x}$ (ii) $1/\lambda$ (iii) $1/\lambda^2$ (iv) $\lambda e^{-\lambda x}$

(c) Let X be a random variable with p.d.f. $f(x) = \frac{1}{2}, -1 \leq x \leq 1$ and $X = Y^2$.The correlation coefficient between X and Y is

(i) 0 (ii) 1 (iii) -1 (iv) -0.5

Q2. Write very short answer for each of the following questions: $6 \times 1 = 6$

(a) Draw the graph of density function of Gamma distribution.

(b) Write the statement of theorem on effect of change of origin and scale on M.G.F.

(c) Find the probability generating function of $Y = X + 1$.

(d) Mention the probability which is given by $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$ if $f(x,y)$ is the joint density function of random variables X and Y.

(e) If one of the regression coefficients is greater than unity how will be the other?

(f) Give the statement of De-Moiver-Laplace's Central limit theorem.

Q3. Write short answer for each of the following: $5 \times 3 = 15$

(a) State and prove Holder's inequality for moments.

(b) A random variable X is Uniformly distributed in (0,1), find the probability function of $Y = X^2 + 1$.

(c) During the period of reduction sale, after the appointment of a particular advertising agency, the sale is distributed exponentially with parameter $\lambda = \frac{1}{2}$. If the sale on 3 days are checked at random what is the probability that on those days the increase is 8 units (Given that $e^{-4} = .0183$).

(d) Draw the normal density curve and give two characterizations depicted by the curve.

(e) Examine whether C.L.T. holds or not if $P(X_k = \pm 2^k) = \frac{1}{2}$.

Q4. Write short answer of the following: $5 \times 4 = 20$

(a) Show that the probability generating function of $Y = \frac{X-a}{b,}$, $b > 0$ is $s^{-a/b} P_X(s^{1/b})$.

(b) If $\emptyset(t)$ is the characteristic function of a random variable X, prove that $\emptyset(t)$ is uniformly continuous.

(c) A daily passenger has taken shelter by the way of his availing the train as soon as the rain stars. If the continuation of rain follows approximately a Gamma variate with parameters $\alpha = 2$ and $\beta = \frac{1}{10}$, and if the rain stops within half an hour, he can avail himself of the train at an eleventh hour. Find the probability that he would get that particular train.

(d) If three uncorrelated variables x_1, x_2, x_3 have the same standard deviations, find the coefficient of correlation between $x_1 + x_2$ and $x_2 + x_3$.

(e) If a random variable $X_r (r = 1,2, \dots, n)$ assumes the values r and $-r$ only, and all $X_r's$ are independent, show that law of large cannot be applied here.

Q5. Answer the following questions: $6 \times 2 = 12$

(a) A random variable X has the following probability function values of X:

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	3k	2k	k^2	$2\,k^2$	$7k^2 + k$

(i) Find the value of k. (ii) Evaluate $P(X < 6)$ and $P(0 < X < 5)$ (iii) Determine the distribution function of X.

OR

"A random variable may not have moments although its M.G.F. exists." Justify the given statement with example.

(a) If X is a random variable which assumes only integral values probability distribution .

$P(X=k)=p_k\ ,k=0,1,2,\ldots$ and $P(X>k)=q_k$ so that $q_k=p_{k+1}+p_{k+2}+\cdots=1-\sum_{i=0}^{k}p_i$ and two generating functions are $P(s)=p_0+p_1s+p_2s^2+\cdots$ and $Q(s)=q_0+q_1s+q_2s^2+\cdots$, prove that for $-<s<1,\ Q(s)=\frac{1-P(s)}{1-s}$.

OR

A random variable X has the exponential distribution given by

$$f(x)=\begin{cases}e^{x}, for\ x>0\\ 0,\ elsewhere\end{cases}$$

Find the probability density of the random variable $Y=\sqrt{X}$. Give the diagram of this transformation.

Q6. Answer any two of the following: $6\times 2=12$

(a) On x-axis (n+1) points are taken independently between the origin and $x=1$, all positions are being equally likely. Show that probability that the $(k+1)^{th}$ of these points, counted from the origin, leis in the interval $x-\frac{1}{2}dx$ to $x-\frac{1}{2}dx$ is $n_{C_k}(n+1)x^k(1-x)^{n-k}dx$.

(b) If random variable X have Beta distribution of first kind with parameters $\alpha>0$ and $\beta>0$, find the mean and variance of X.

(c) The mean yield for one-acre plot is 662 kilos with a s.d.32 kilos. Assuming normal distribution how many one-acre plots in a batch of 1,000 plots would you expect to have yield (i) over 700 kilos, (ii) below 650 kilos,.[Given P(0<Z<1.19)=0.383 and P(0<Z<0.38)= 0.148]

Q7. Answer any one of the following: $6\times 2=12$

(a) Show that if X_1 and X_2 are standard normal variates with correlation coefficient ρ between them, then the correlation coefficient between ${X_1}^2$ and ${X_2}^2$ is given by ρ^2.

(b) State and prove Chebychev's inequality.

(c) Establish an expression for Moment generating function of bivairate normally distributed random variables X and Y.
